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STUDY OF CROSSED-FIELD
AMPLIFIERS

Progress Report
16 March through 15 June, 1966

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UNITED STATES ARMY ELECTRONICS COMMAND · FORT MONMOUTH, N.J.

Contract DA36-039 AMC-02164(E)

ELECTRONICS RESEARCH LABORATORY
College of Engineering
University of California, Berkeley

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STUDY OF CROSSED-FIELD AMPLIFIERS

Twelfth Quarterly Progress Report

16 March through 15 June 1966

Contract No. DA 36-039 AMC-02164(E)

DA Project No. 3A99-13-001

Prepared by

Electronics Research Laboratory
College of Engineering
University of California, Berkeley

for

U. S. Army Electronics Command, Fort Monmouth, N. J.

ABSTRACT

The design of a crossed-field gun, in which the cathode and accelerating regions are shielded from the magnetic field, is completed. The geometry of the magnetic shield and focusing electrodes of the shielded gun which has a Brillouin flow characteristic at the gun exit is specified. The computer programs developed for a second order paraxial-ray crossed-field beam synthesis is now operational.

Crossed-field effects on the potential minimum stability are being investigated using a space-charge feedback model for computer calculations. Numerical results indicate that instability takes place when the cathode length equals or exceeds 0.6 of a cycloid length.

A dispersion relation for a crossed-field interaction between drifted carriers and a macroscopic EM field in a solid has been derived. Investigation of the dispersion relation so far has not uncovered any instability within the physical parameters used.

STUDY OF CROSSED-FIELD AMPLIFIERS
Twelfth Quarterly Progress Report

Work continues under this contract on noise effects in crossed-field devices, on the design of crossed-field guns, and on the exploration of crossed-field interactions in solids. Two phases of the work on gun design were completed during this quarter, both related to the method of crossed-field gun design using the paraxial ray equations with computers. The study of noise effects near the cathode continued with additional computer solutions and with the design and construction of a new experimental tube that will be tested during the following quarter. The effort on crossed-field interactions on solids was concentrated on solving a basic dispersion relation with approximations pertinent to a linearly polarized (Alfven) wave. The numerical solutions for the parameters chosen did not appear encouraging, so other parameters or other approximations will be made in following work.

Dr. A. Rao is completing work with the University at this time. Mr. K. Weller is beginning active work on the project.

SHIELDED-GUN LOW-NOISE AMPLIFIER
(R. A. Rao, Professor T. Van Duzer and Dr. S. P. Yu)

The project has been terminated and the results are published as a Ph. D. thesis entitled "A Synthesis Method for Crossed-Field Electron Guns." An abstract of the thesis follows.

A general approximate method for synthesizing crossed-field sheet beams and determining electrodes to produce such beams is developed. The limitations of the method and its accuracy are discussed with reference to some problems of practical importance in microwave amplifiers.

The problem of synthesizing an electron beam requires compromises among various competing factors; hence it is desirable to work with approximate methods in which the effects of the various requirements may be studied and modifications may be made on these requirements. The first-order paraxial ray equation is one in which the important beam parameters are included in a simple mathematical description of the central trajectory of the beam. The approximation comes about in specifying the beam parameters on the nonaxial trajectories in terms of power series expansions of their values on the axis and neglecting higher order terms in beam thickness. This approximation is shown to be sufficiently accurate for thin beams.

The numerical solutions for the beam parameters obtained from the paraxial ray equation may be put in analytic form by using polynomial approximations, and analytic methods may be used for determining the electrodes to produce such beams. The method used here to determine electrodes makes use of conformal transformations to transform the beam edges into the real axis of a complex plane and determines the equipotentials in this plane by analytic continuation of a suitable complex potential function. The equipotentials are finally retransformed into the original plane of the problem to obtain the electrodes. Extensive use has been made of the IBM 7094 digital computer for obtaining numerical solutions.

An electron beam with Kino short-gun characteristics near the cathode and Brillouin-flow characteristics in the drift region is synthesized as an example for the use of the method. A carefully planned experiment using this design showed that the beam characteristics in the experimental tube are in good agreement with the theoretical predictions.

An electron beam emanating from a strip cathode inside a hollow magnetic spheroid and having Brillouin-flow characteristics in the drift region outside the spheroid is synthesized as a second application of the synthesis method. Two such beams and the electrodes to produce such beams are designed. Some suggestions for improving the accuracy of the method are given.

CATHODE REGION STUDIES
(R. Y. C. Ho, Professor T. Van Duzer, Dr. S. P. Yu)

The objective of this project is to construct a relatively simple space-charge feedback model to study the effects of the space charge and the magnetic field on the potential minimum perturbation so as to explain the noise phenomena under the space-charge limited condition in both long and short guns. The effects on noise are expressed in terms of shot-noise smoothing factors.

The space charge feedback model is being refined. It is constructed as follows. Consider a crossed-field diode originally operating under space-charge-limited conditions with a potential minimum V_m at a distance x_m from the cathode. We shall assume that the potential distribution is parabolic in the region between the cathode and the potential minimum and is given by $\phi = X(1 + X/3)$ beyond the potential minimum where ϕ is the normalized potential, X is the normalized distance measured from the potential minimum. We note that the above potential distribution is an analytic approximation of the equipotentials from Rao's crossed-field gun design.[1] Now assume that starting from some instant, an excess of emission current (shot noise) is emitted from the cathode which is assumed to have only __32__ velocity classes. And further assume that all the electrons leaving the cathode belong to these __32__ velocity classes. With these assumptions we will then only have a finite number of critical initial velocity classes, and hence a finite number of critical planes associated with them. Only those electrons in the critical initial velocity classes will be affected by the introduction of shot-noise. The introduction of shot-noise (excess emission current) causes an increase of the space charge so that it deepens the potential minimum V_m. The potential minimum perturbation δV_m will cause the critical electrons to flow back to the cathode which otherwise would go into the anode region. We simulate the perturbed critical electrons by an electron flow from its critical plane, to

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the cathode and a hole flow, which is positive charged, to compensate
for the lack of the electrons in the region beyond the potential minimum
in the perturbed state. We use the line charge model and take into
account the imaging of the charges by the cathode. We can determine
the Green's functions, which relate the current from a point of the
cathode to the potential minimum perturbation δV_m at a point in front
of the cathode. The total perturbation of the potential minimum at a
point in front of the cathode may be written as

$$\delta V_m = \delta V_{ms} + W \sum_{\ell_k} G_{h-e} \tag{1}$$

where δV_{ms} is the potential minimum perturbation due to the shot
noise alone, G_{h-e} is the net Green's function due to the holes and the
critical electrons, and W is a weighting function. The effects of the
cathode length and the magnetic field on shot noise are considered
separately.

1. Effects of Cathode Length

The summation of the absolute values of the Green's functions
is shown as a function of the cathode length in Fig. 1. In this particular
case, the weighting function $\underline{W}$ takes the value of 1.5. When the second
term of Eq. (1) is larger than the absolute value of the first term,
δV_m is larger than zero and hence the beam current increases. This
situation signifies a potential-minimum instability. The shot-noise
smoothing factor Γ^2 is shown as a function of the cathode length in
Fig. 2. It is seen that when $\ell_k/\ell_c \cong 0.54$ the instability occurs. We
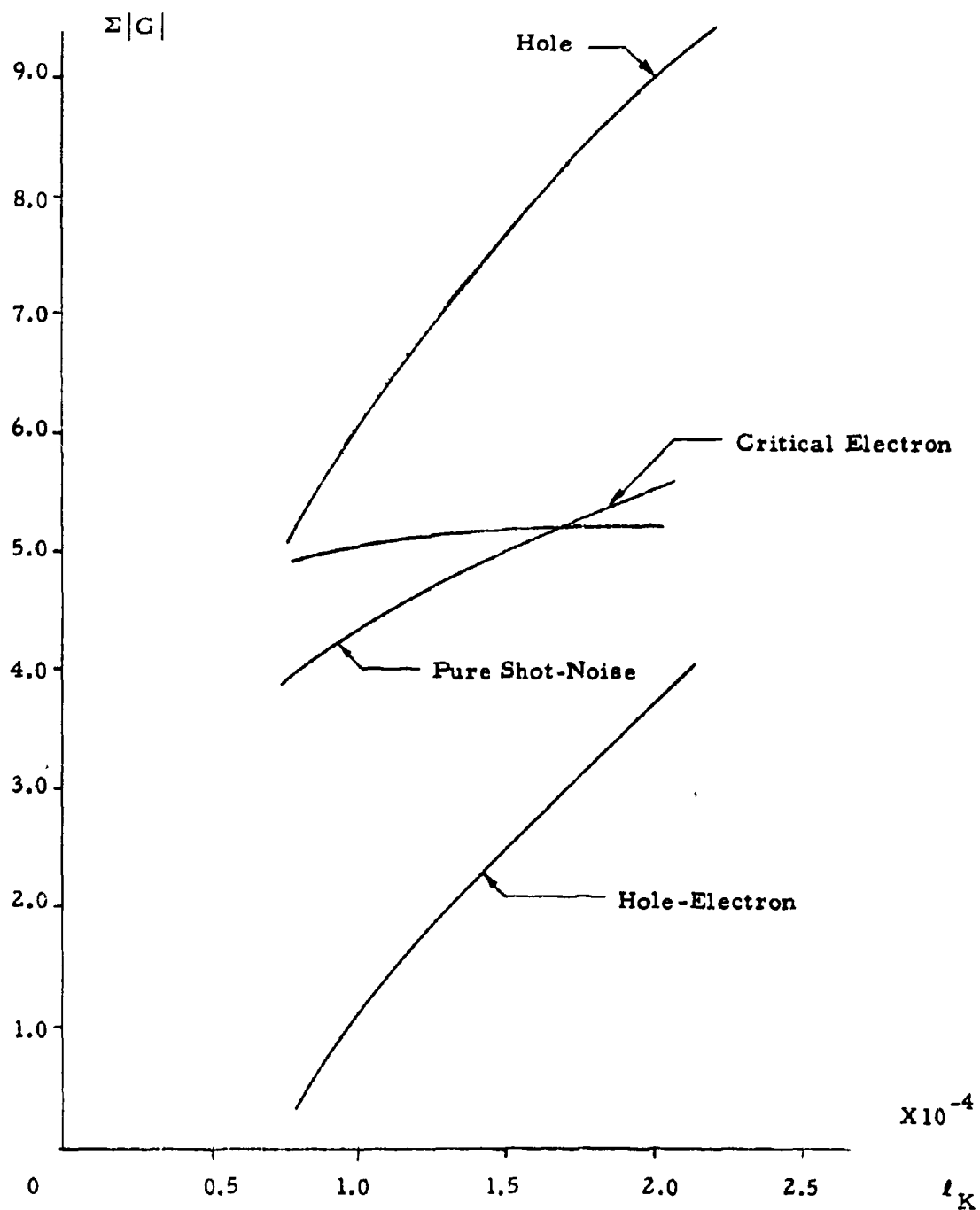note that ℓ_c is the cycloid length.

Fig. 1. Average value of the summation of the absolute value of Green's functions vs cathode length for B = 600 Gauss.
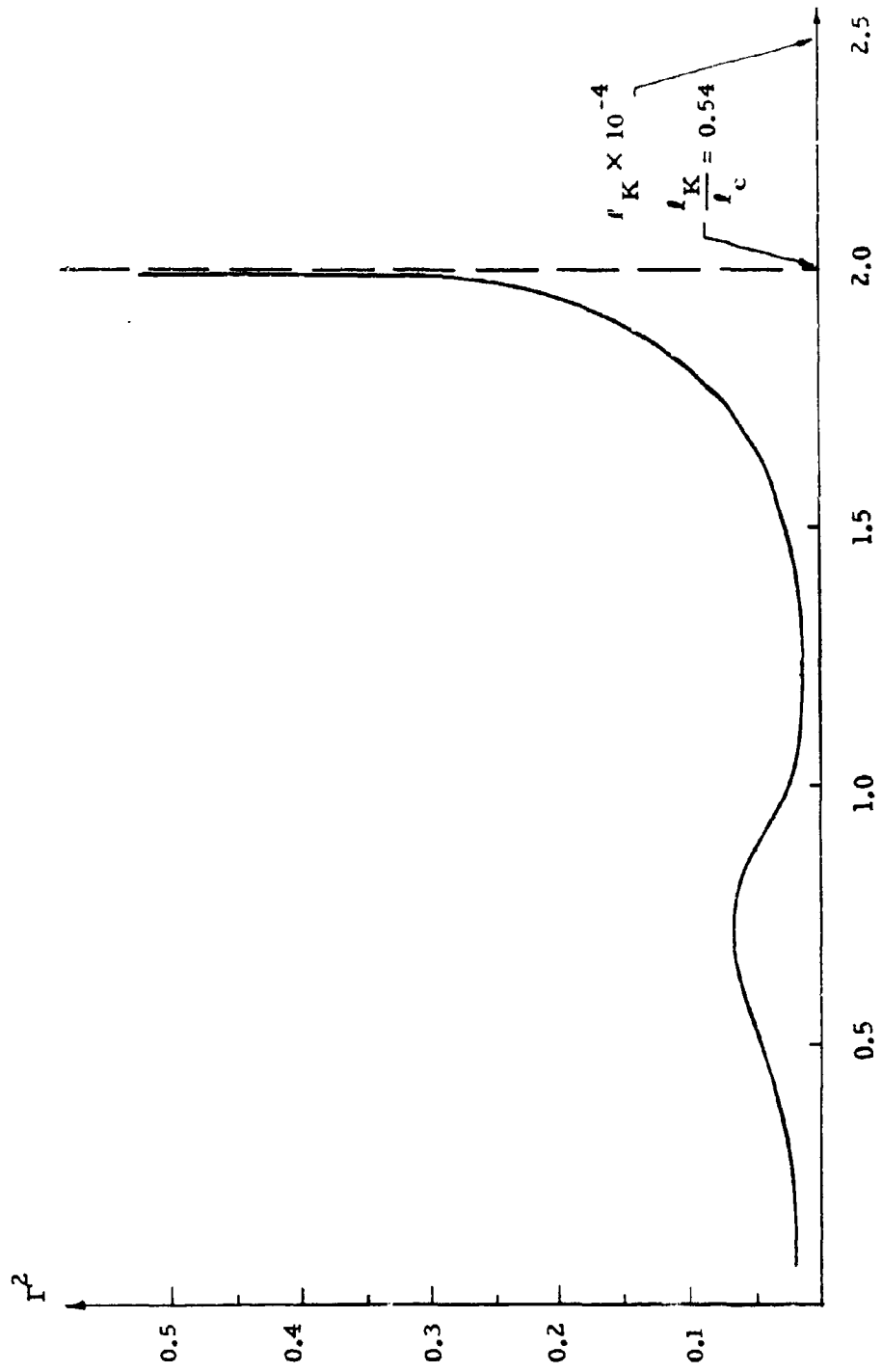
Fig. 2. Shot-noise factor vs cathode length for B = 600 gauss.

2. Effect of Magnetic Field

The summation of the absolute values of the Green's functions is shown as a function of the magnetic field in Fig. 3. It is seen that the potential minimum instability occurs when B = 800 gauss which, in this model, corresponds to $l_k/l_c \cong 0.6$. The shot-noise smoothing factor Γ^2 is shown as a function of the magnetic field in Fig. 4.

As a continuation of this project we are planning: (1) to locate the critical transition of the shot noise smoothing factor Γ^2 experimentally by measuring the noise figures of a crossed-field gun for different cathode lengths, say, $l_k/l_c = 0.8$ and 0.5; (2) to calculate value of the Γ^2 as a function of the space-charge-limited condition; (3) to check the validity of the model by changing the potential distribution and studying its effect on the onset of potential minimum stability. The tube designed for the experimental part of the program is now ready for testing.

CROSSED-FIELD-TYPE INTERACTIONS IN SOLIDS
(M. Meyer, Dr. S. P. Yu, Professor J. R. Whinnery)

In the last report we described a crossed-field interaction between the current carriers in a solid conductor and a macroscopic EM field. The purpose of this study is to look for possible instabilities which can be used to generate or amplify microwaves.

Using the model previously described we obtain the following dispersion relation

$$k^2 = \frac{\omega^2}{c^2} \, \epsilon^1 - \sum_{i=e,h} \frac{j\omega_{p_i}^2 \, (\omega - kv) \, \tau_i \, [1 + j(\omega - kv)\tau_i]}{(\omega_{c_i}\tau_i)^2 + [1 + j(\omega - kv)\tau_i]^2} \tag{1}$$
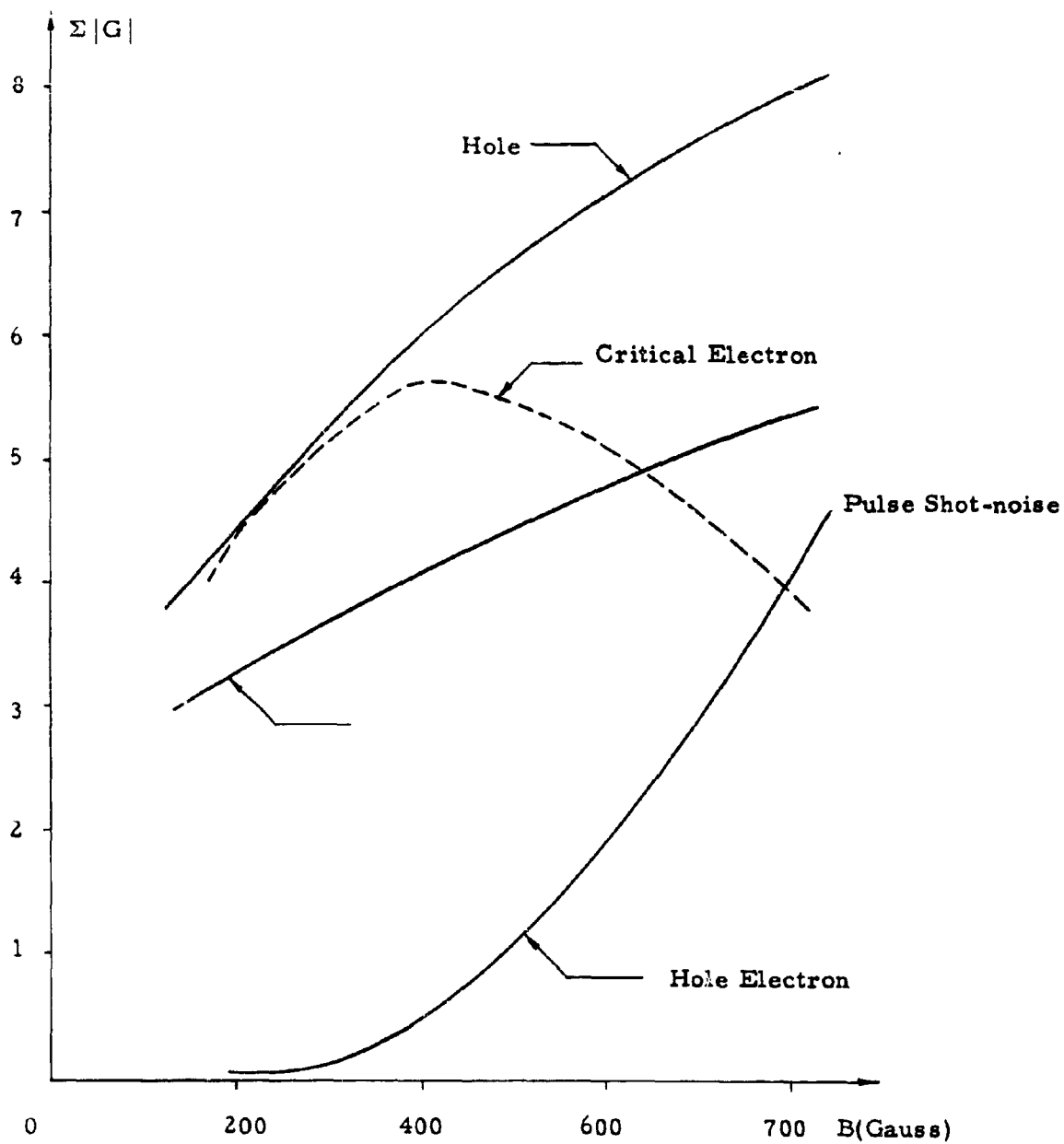
Fig. 3. Average value of the summation of the absolute value of the Green's functions vs magnetic field, for $l_K = 1.2 \times 10^{-4}$ m.
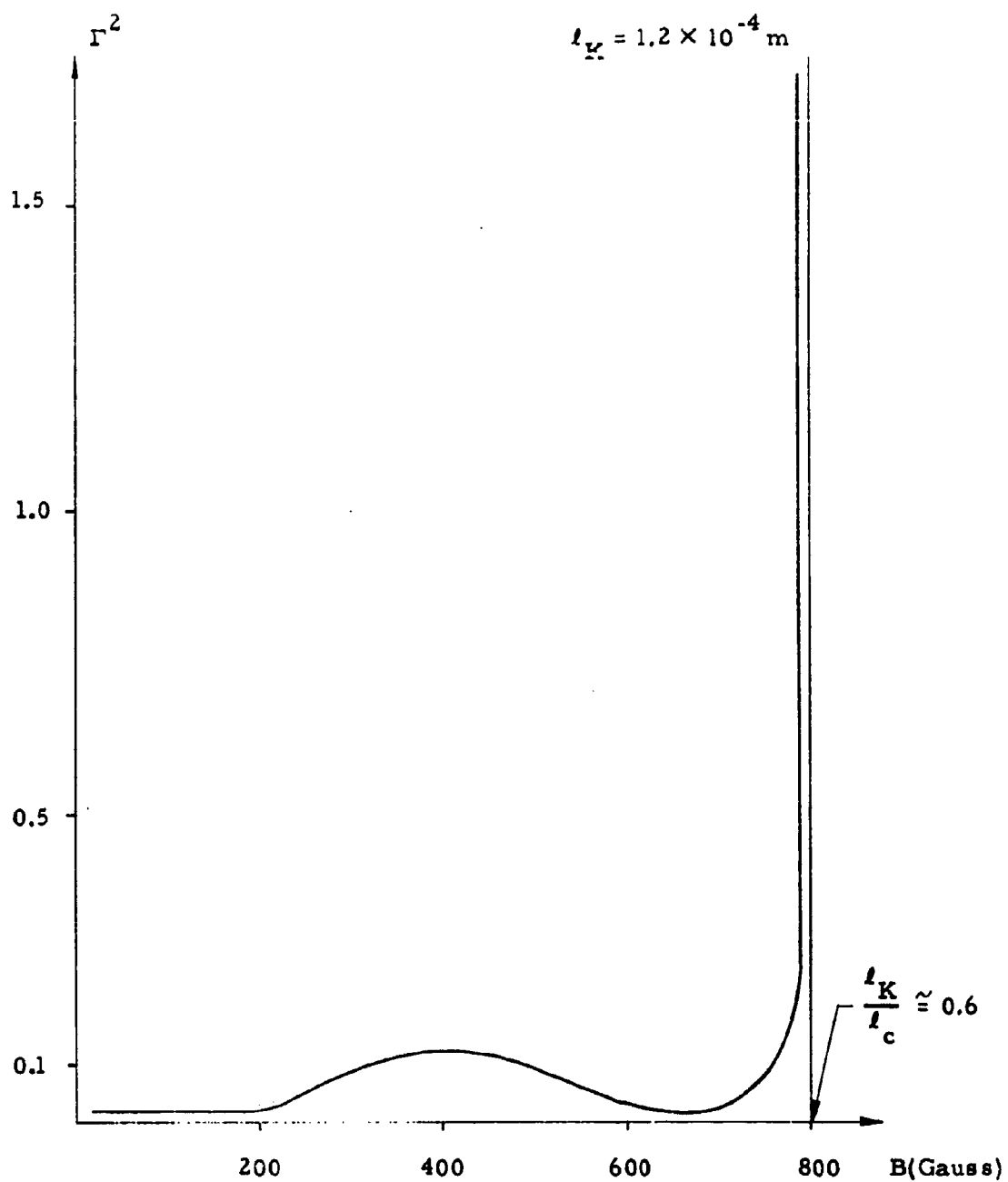
Fig. 4. Shot-noise factor vs magnetic field for $\ell_K = 1.2 \times 10^{-4}$ m.

and two of the relevant terms in the dielectric tensor $\hat{\epsilon}$

$$\alpha_{21} = +(1 - \frac{k}{\omega}v) \sum_{i=e,h} j \left(\frac{\omega_{p_i}}{\omega}\right) \frac{(\omega_{c_i}\tau_i)(\omega_{p_i}\tau_i)}{(\omega_{c_i}\tau_i)^2 + [1 + j(\omega - kv)\tau_i]^2} \tag{2}$$

$$\alpha_{22} = \epsilon^1 - \sum_{i=e,h} j \left(\frac{\omega_p}{\omega}\right) \frac{[1 + (\omega - hv)\tau_i](\omega_{p_i}\tau_i)}{(\omega_{c_i}\tau_i)^2 + [1 + j(\omega - kv)\tau_i]^2}. \tag{3}$$

The sums are over the two carrier types, electrons and holes. The above dispersion equation has the following important built-in assumptions. First, $\omega_{c_i}\tau \gg 1$ for electrons and holes; this allows use of one velocity $\bar{v} = \bar{E}_0 \times \bar{B}_0/B_0^2$ for both carrier types. Second, we assume $|\alpha_{22}| \gg |\alpha_{21}|$ which for the given model guarantees a linearly polarized (Alfven) wave. If we say $\omega_c\tau \gg 1$ for one carrier type only, it then becomes impossible to satisfy reasonable conditions whereby $|\alpha_{22}| \gg |\alpha_{21}|$. There are two conditions under which $|\alpha_{22}| \gg |\alpha_{21}|$. One is $\omega = kv$, in which case $\alpha_{21} = 0$ and $k^2 = \frac{\omega^2}{c^2} \epsilon^1$. The other requires that $n = p$ and $\omega_c\tau \gg \omega\tau \gg 1$ with no restriction on v. In the case where $v = \omega/k$ and $k^2 = \frac{\omega^2}{c^2} \epsilon^1$ we see that $v = c/\sqrt{\epsilon^1}$ which, considering that $\epsilon^1 = 16$ is typical, is much faster than carriers in a solid can travel. If $\omega_c\tau \gg \omega\tau \gg 1$ and $n = p$, then the two terms in α_{21} will be equal in magnitude and opposite in sign and will therefore make α_{21} equal zero. This would not be the case if v were not the same for electrons and holes; hence crossed-fields are necessary to maintain an Alfven wave in the presence of drifted carriers. It would require a rather high frequency to make $\omega\tau \gg 1$ in a semiconductor because

$\tau = 10^{-11}$ sec is about the largest value that can be expected. A semi-metal such as bismuth would be more desirable because of the relatively large τ that can be obtained at low (4°K) temperatures. For $\omega_c \tau \gg \omega\tau \gg 1$, we obtain

$$k^2 = \frac{\omega^2}{c^2} \left[\epsilon^1 - (1 - 2\frac{kv}{\omega})\left(\frac{\omega_{p_e}^2}{\omega_{c_e}^2} + \frac{\omega_{p_h}^2}{\omega_{c_h}^2}\right) \right] \tag{4}$$

where it was assumed that $kv/\omega \ll 1$, as would be the case for reasonable values of v. This dispersion relation does not lead to an instability.

It was hoped that a complete investigation of Eq. (1), using a computer, would reveal experimentally workable regions where an instability could occur. Because of the many parameters involved, excessive computer time would be required to exhaust all the possibilities; therefore restricted parameter ranges were chosen to correspond to likely experimental conditions. Only a few results have been obtained but they are discouraging enough to question whether further analysis should be made on this particular model. However, the investigation has not been so extensive as to rule out other possible crossed-field interactions in solids. Other such interactions are currently being investigated.

SYNTHESIS OF BEAMS FOR CROSSED-FIELD AMPLIFIER
(G. A. Poe, Professor T. Van Duzer, and Dr. S. P. Yu)

The objective of this project was to evaluate second-order corrections to the solution of the first-order paraxial-ray equation. This work has been terminated and the latest results are reported in the following.

Tl з synthesis of sheet electron beams flowing in crossed-electric and magnetic fields has been possible when the beam is sufficiently "thin" because the first-order paraxial-ray approximation could be made. Whenever "thick" beams are studied, however, the first-order paraxial ray model may no longer be accurate enough. For this reason, a second-order paraxial ray model together with two computer programs have been developed for "thick" beam studies. The work on the second-order paraxial ray model and the results of a "thick" beam calculation can be found in previous quarterly reports. The current report deals with the functions of the two computer programs.

The computer programs, designated programs A and B, are designed to be used after a first-order flow has been developed. Both programs require the same form of data input. Values of the first-order scaling factor, ξ_1, the potential, ϕ, and the curvature, K, and their derivatives ξ_1', ξ_1'', and ϕ' must be known before either of the programs can be executed. Program A is for solving the second-order paraxial ray equation in which the unknown quantity is ξ_q. The second-order beam thickness at any distance, s_1, along the beam is then given as

$$\xi(s_1) = \xi_1(s) + \xi_q(s)q_2,$$

where q_2 is the coordinate transverse to the beam flow. When the quantity $\xi(s_1)$ is known, program B may be used to determine the electric field components along the upper and the lower edges of the beam. The knowledge of these field components is essential in the synthesis of the electrodes which produce the desired electron flow.

The programs require the number of data points along the $q_2 = 0$ line to be less than or equal to 71 and greater than or equal to 7, denoted by NN. The data are to be read-in in equidistant intervals denoted by

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H. The first point where data are given is denoted by S1. The half-cathode width is denoted Q2 and is positive.

The programs use interpolation techniques[i] to divide the given input interval into 20, new, equidistant intervals.

After the data have been read-in and interpolated, differentiation techniques[1] are employed to find values of K', K'', ϕ'' and ξ_1'''. The second-order differential equation in ξ_q is solved by reducing the equation to two simultaneous first-order differential equations and employing a modified Runge-Kutta method.[2]

The output for program A, the second-order beam-width calculation program, is presented in a table with headings as follows: Arc Length, Upp HBeamwidth, Low HBeamwidth, Old HBeamwidth, Eq, Eqd, Eqdd, and Potential. The output would have the above eight headings across the page and values following directly under.

The title Arc Length is for the $q^2 = 0$ coordinate arc length.

The title Upp HBeamwidth is the upper half second-order beamwidth.

The title Low HBeamwidth is the lower half second-order beamwidth.

The title Old HBeamwidth is the first-order half beamwidth which is symmetric about the $q_2 = 0$ trajectory.

A block diagram of the over-all operation of program A is as shown in Fig. 5. To calculate the beamwidths one merely arranges the input data in the following order:

    (1) Place NN, H, S1, Q2 in FORMAT (I5, 3F10.6)

    (2) Place P(I), the Potential, in FORMAT (E14.7, 3E15.7)

    (3) Place PD(I), $\frac{dP(I)}{dS1}$, in FORMAT (E14.7, 3E15.7)

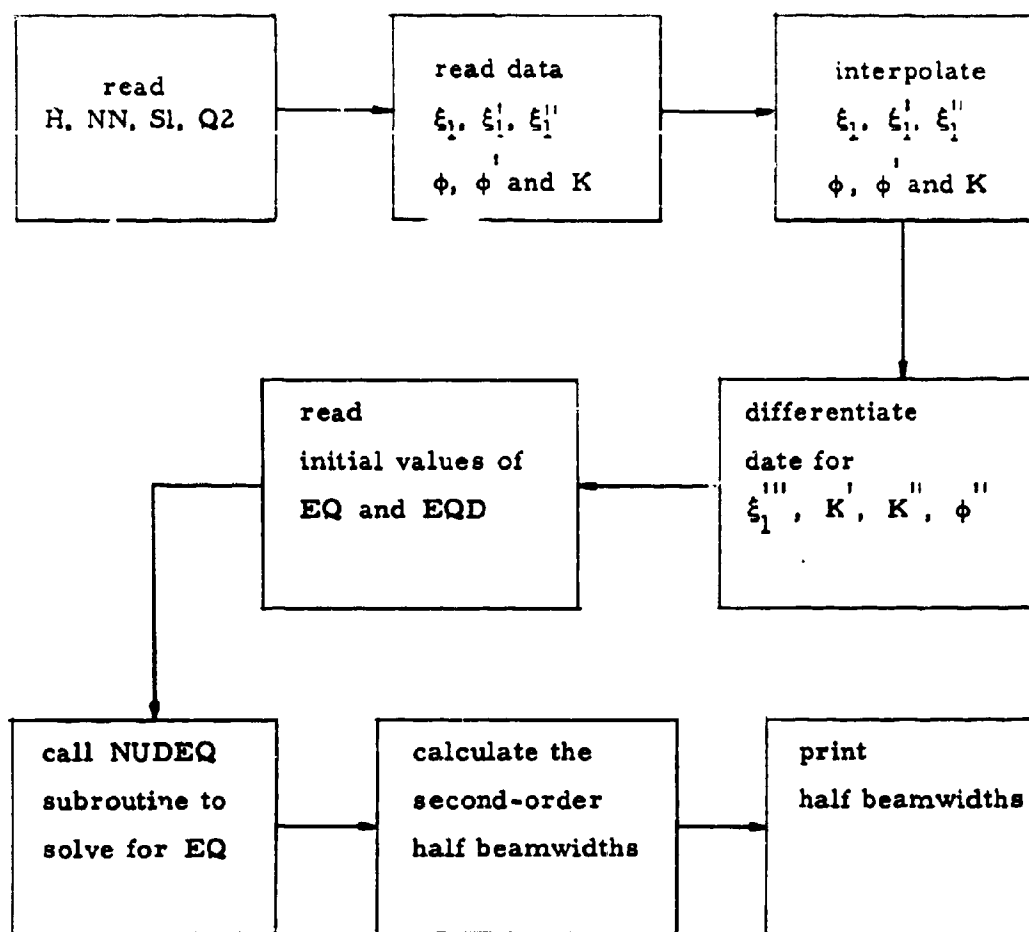   (4) Place E(I), the first-order scale factor, in same FORMAT as in (2) and (3)

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Fig. 5. Program A.

(5) Place ED(I), $\dfrac{dE(I)}{dS_1}$ in FORMAT as above.

(6) Place EDD(I) $\dfrac{dED(I)}{dS_1}$ in FORMAT as above.

(7) Place C(I), the curvature in FORMAT as above.

(8) Place EQ and EQD in FORMAT (2E15.7)

With the above arrangement of the input information, the computer program will calculate and print out the first-order and second-order beamwidths.

To find the beam edge electric fields, one uses the same input data as above in the same form and order but with computer program B. The output from program B will be values of the normal and tangential components of the electric field for the upper and lower beam edges. These quantities appear in the output of program B as follows: Arc Length, EFTU, EFNU, EFTL, EFNL, EQ.

The title EFTU means the electric field tangential to the upper beam.

The title EFNU means the normal electric field of the upper beam.

The title EFTL means the tangential electric field to the lower beam.

The title EFNL means the normal electric field of the lower beam edge. EQ is the second-order scale factor.

The values of the above quantities appear immediately under the titles in the output.

The computer programs developed to calculate the beamwidths and the data used in calculating the beam edge electric fields can be applied to any crossed-field sheet beam problem. After the first-order flow has been developed, the second-order flow parameters can be calculated.

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1. James B. Scarborough, Numerical Math. Analysis , The John Hopkins Press: 1962, Baltimore.

2. S. Gill, "A Process of Step-by-Step Integration of Differential Equations on Automatic Digital Computing Machine," Proc. Camb. Phil. Soc., 47 (1951) pp. 96-108.

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